Slides
Condensed Matter Physics
Revision Lecture 1

HOMEWORKS:

Go to my website for commentary

Much of this problem set is standard exam material.

Problem Set 1

Einstein, Debye, Drude, and Free Electron Models

1.1. Einstein Solid

(a) **Classical Einstein Solid (or "Boltzmann" Solid):**

The classical calculation has never been on an exam (although it is examinable)

Consider a single harmonic oscillator in three dimensions with Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} + \frac{k}{2}\mathbf{x}^2$$

▷ Calculate the classical partition function

$$Z = \int \frac{\mathbf{dp}}{(2\pi\hbar)^3} \int \mathbf{dx}\, e^{-\beta H(\mathbf{p},\mathbf{x})}$$

Note: in this problem $\mathbf{p}$ and $\mathbf{x}$ are three dimensional vectors (they should appear bold to indicate this unless your printer is defective).

▷ Using the partition function, calculate the heat capacity $3k_B$.

▷ Conclude that if you can consider a solid to consist of N atoms all in harmonic wells, then the heat capacity should be $3Nk_B = 3R$, in agreement with the law of Dulong and Petit.

(b) **Quantum Einstein Solid:** Now consider the same Hamiltonian quantum mechanically.

▷ Calculate the quantum partition function

$$Z = \sum_j e^{-\beta E_j}$$

The quantum Einstein model could be on an exam (although it rarely is)

where the sum over j is a sum over all Eigenstates.

▷ Explain the relationship with Bose statistics.

▷ Find an expression for the heat capacity.

▷ Show that the high temperature limit agrees with the law of Dulong of Petit.

▷ Sketch the heat capacity as a function of temperature.

Debye theory is very frequently examined!

1.2. **Debye Theory:**

(a)‡ State the assumptions of the Debye model of heat capacity of a solid.

▷ Derive the Debye heat capacity as a function of temperature (you will have to leave the final result in terms of an integral that cannot be done analytically).

▷ From the final result, obtain the high and low temperature limits of the heat capacity analytically.

You may find the following integral to be useful
$\int_0^\infty dx \frac{x^3}{e^x-1} = \sum_{n=1}^\infty \int_0^\infty x^3 e^{-nx} = 6\sum_{n=1}^\infty \frac{1}{n^4} = \frac{\pi^4}{15}$
By integrating by parts this can also be written as $\int_0^\infty dx \frac{x^4 e^x}{(e^x-1)^2} = \frac{4\pi^4}{15}$

(b) The following table gives the heat capacity C for potassium iodide (KI) as a function of temperature.

T(K)	0.1	1.0	5	8	10	15	20
C (J K $^{-1}$ mol $^{-1}$)	8.5×10^{-7}	8.6×10^{-4}	1.2×10^{-1}	5.9×10^{-1}	1.1	2.8	6.3

▷ Discuss, with reference to the Debye theory, and make an estimate of the Debye temperature.

1.3. Drude Theory of Transport in Metals

Drude theory does show up on exams -- particularly in the context of semiconductors

(a) Assume a scattering time τ and use Drude theory to derive an expression for the conductivity of a metal.

(b) Define the resistivity matrix $\underset{\sim}{\rho}$ as $\vec{E} = \underset{\sim}{\rho}\vec{j}$.

▷ Use Drude theory to derive an expression for the matrix $\underset{\sim}{\rho}$ for a metal in a magnetic field. (You might find it convenient to assume $\vec{B}$ parallel to the $\hat{z}$ axis. The under-tilde notation means that the quantity $\underset{\sim}{\rho}$ is a matrix.)

▷ Invert this matrix to obtain an expression for the conductivity matrix $\underset{\sim}{\sigma}$.

(c) Define the Hall coefficient.

▷ Estimate the magnitude of the Hall voltage for a specimen of sodium in the form of a rod of rectangular cross section 5mm by 5mm carrying a current of 1A in a magnetic field of 1T. The density of sodium atoms is roughly 1 gram/cm^3, and sodium has atomic mass of roughly 23. You may assume that there is one free electron per sodium atom (Sodium has *valence* one).

▷ What practical difficulties would there be in measuring the Hall voltage and resistivity of such a specimen (and how might these difficulties be addressed).

(d) What properties of metals does Drude theory not explain well?

AC Drude theory is likely to be too hard for an exam

(e)* Consider now an applied AC field $\vec{E} \sim e^{i\omega t}$ which induces an AC current $\vec{j} \sim e^{i\omega t}$. Modify the above calculation (in the presence of a magnetic field) to obtain an expression for the complex AC conductivity matrix $\underset{\sim}{\sigma}(\omega)$. For simplicity in this case you may assume that the metal is very clean, meaning that $\tau \to \infty$, and you may assume that $\vec{E} \perp \vec{B}$. You might again find it convenient to assume $\vec{B}$ parallel to the $\hat{z}$ axis. (This problem might look hard, but if you think about it for a bit, it isn't really much harder than what you did above!)

▷ At what frequency is there a divergence in the conductivity?

▷ What does this divergence mean? (When τ is finite, the divergence is cut off).

▷ Explain how could one use this divergence (known as the cyclotron resonance) to measure the mass of the electron. (In fact, in real metals, the measured mass of the electron is generally not equal to the well known value $m_e = 9.1095 \times 10^{-31}$ kg. This is a result of *band structure* in metals, which we will explain later in the course.)

ETC…. SEE MY WEBSITE

Sample Exams with Solutions:

One on my website

One in back of book

Solutions to 2011 exam on web – with all the past papers

PAST PAPERS (1996- present)

Go to my website
for commentary

Which Past Paper Questions are on the Syllabus Now?

Condensed Matter Papers 2011 and thereafter. All on Syllabus

B Paper 2010

Q1. On Syllabus. End of part d is tricky and was not really covered, but could be deduced by a perceptive student.

Q 2,3,4,5. On Syllabus.

Q6. Mostly not on syllabus. The students should be able to deduce the density of states of a 2d electron gas.

Q7,8 Not on syllabus

ETC !!!!
SEE MY WEBSITE!

B Paper 2009.

Q1. On syllabus. This question is solved in great detail in my lecture notes (note also there is an error in the height of data point e of the plot. Discussed in the book)

Q2,3. On syllabus.

STATISTICAL ANALYSIS
OF PAST PAPERS (2004-present)

Go to my website
for commentary

Topic Subtopic	Year =	04	05	06	07	08	09	10	11	12	13	14
	# of Times											
Something About Phonons	11	1	1	1	1	1	1	1	1	1	1	1
Define Phonon	1	1										
Phonon Density of States	2						1					1
In 2d	1						1					
In 1d / diatomic	1				1							
How would you measure phonons (light/neutrons)	2		1		1							
Why is there a degeneracy of modes at...	2		1					1				
Debye Specific Heat	4		1	1			1			1		
Derivation in 3d	3			1						1		1
Derivation In 2d	2		1				1					
Derivation In 1d	1			1								
How many/ what kind of (acoustic/optical/transverse/longitudinal) phonon mod	5				1	1	1	1		1		
Describe Motion of acoustic/optical modes	4	1			1	1	1					
Some Sort of Harmonic Chain	7		1	1	1	1		1	1		1	
Diatomic with Two Masses	2				1	1						
Monatomic	3			1					1		1	
Alternating Sprint Constants	2		1					1				
Second or Further Neighbor interactions	1										1	
monatomic limit of diatomic	2		1			1						
Sketch Dispersions / monotomic diatomic	2	1							1			
Something about the Free Electron Gas	8		1		1	1	1	1		1	1	1
Derive Specific Heat of Fermi Gas	2		1		1							
Define Fermi Energy / Fermi Surface	3					1		1				1
Density of States of Free Electron Gas	3		1			1		1				
Definition of	1					1						
Derivation In 3d	1							1				
Derivation In 2d	2		1			0.5		0.5				
Derivation In 1d	0.5						0.5					
Estimate a Fermi Energy / Relationship of N to Ef	6		1		1		1			1	1	1

Topic Subtopic	Year =	04	05	06	07	08	09	10	11	12	13	14
	# of Times											
Something About Diffraction / Crystal Structure	11	1	1	1	1	1	1	1	1	1	1	1
Derive Structure Factor / Scattering Amplitude	6	1	1			1		1	1		1	
Calculate Interplanar distances	3		1							1	1	
Diffraction	7	1				1	1	1	1		1	1
Derive Systematic Absences	3							1	1			1
When two atoms scatter same; H not scattering	2			1					1			
Analyze a Powder Diffraction Pattern	5	1			1		1			1	1	
Predict Diffraction Data	2			1		1						
Write Down Structure Factor for X	4					1	1		1			1
Identify a unit cell doubling	2	1	1									
Plan View	2					1		1				
primitive vs conventional unit cell	4			1		1	1	1				
Identify Lattice/Basis	4			1		1				1	1	
Calculate Reciprocal Lattice	2	1	1									
Wigner Seitz / Brillouin Zone Construction	3	1						1		1		
Contrast neutron/xray	1						1					
Describe equipment for neutron/xray	2	1	1									

Topic Subtopic	Year =	04	05	06	07	08	09	10	11	12	13	14
	# of Times											
Something about Band Structure/Semiconductor Physics	10	1	1	1	1	1	1	1	1	1		1
Nearly Free Electron Model (NFEM)	6			1		1	1	1	1	1		
Derive Gaps of NFEM at zone boundary	3					1		1	1			
Draw Dispersion	2						1	1				
Describe Effective Mass	3					1		1		1		
Monovalent / Divalent - Metal/Insulator	3					1	1	1				
Gaps open when doubling unit cell	1						1					
Draw a fermi surface in 2d/3d for weak/strong potential	2					1			1			
Tight Binding Band	1			1								
Describe Density of States	1			1								
Describe opening of gap	1			1								
Define Effective Mass	4	1				1	1					1
Define Chemical Potential / Doping	1					1						
Define Mobility	4	1				1	1					1
Define Conductivity	1						1					
Define Hole	2		1									1
Signs of velocity, energy, current, ...	2		1									1
Law of Mass Action / formula for n(T,mu)	5		1		1	1		1		1		
Derivation	4				1	1		1		1		
Use to calculate some density/mu when doped	4		1		1			1		1		
Temperature dependence of semiconductors	2	1				1						
Estimate band gap / doping from data	1					1						
How this would be measured	2	1				1						
How chemical potential changes with doping	1		1									
Density of States (1d, 2d, 3d)	2				0.5	0.5	0.5	0.5				
Optical Properties of Semiconductors	1						1					
Direct / Indirect Gap	1						1					
States bound to donors	1						1					
Drude Theory	2						1					1
Derive Hall Coefficient	1						1					
Derive Conductivity/Mobility	2	1					1					
Extract mobility/density from experimental data	2						1					1

Topic Subtopic	Year =	04	05	06	07	08	09	10	11	12	13	14
	# of Times											
Something about magnetism	9	1	1	1	1	1		1	1		1	1
Define Para/Diamagnetism	5			1		1		1			1	1
Estimate Larmor Diamagnetism	1			1								
General Curie Law Derivation	1										1	
Curie Law Derivation for Spin 1/2	4			1		1		1				1
Derive Pauli Paramagnetism	1					1						
Adiabatic Demagnetization	1							1				
What is exchange J	2	1			1							
Molecular (mean) field	6	1	1	1	1				1			1
Relationship of J to Tc	3		1	1	1							
What causes domains	1	1										
Domain Relation to Hysteresis	2	1			1							
Derive Size of Bloch Wall	1	1										

The syllabus

VI: Condensed-matter physics

Free electron model of metals, Fermi energy and Fermi surface. Drude theory, conductivity and Hall effect (one carrier only).

Lattice vibrations: law of Dulong and Petit; phonons; dispersion relation with two atomic types: acoustic and optical branches; Einstein and Debye models of heat capacity.

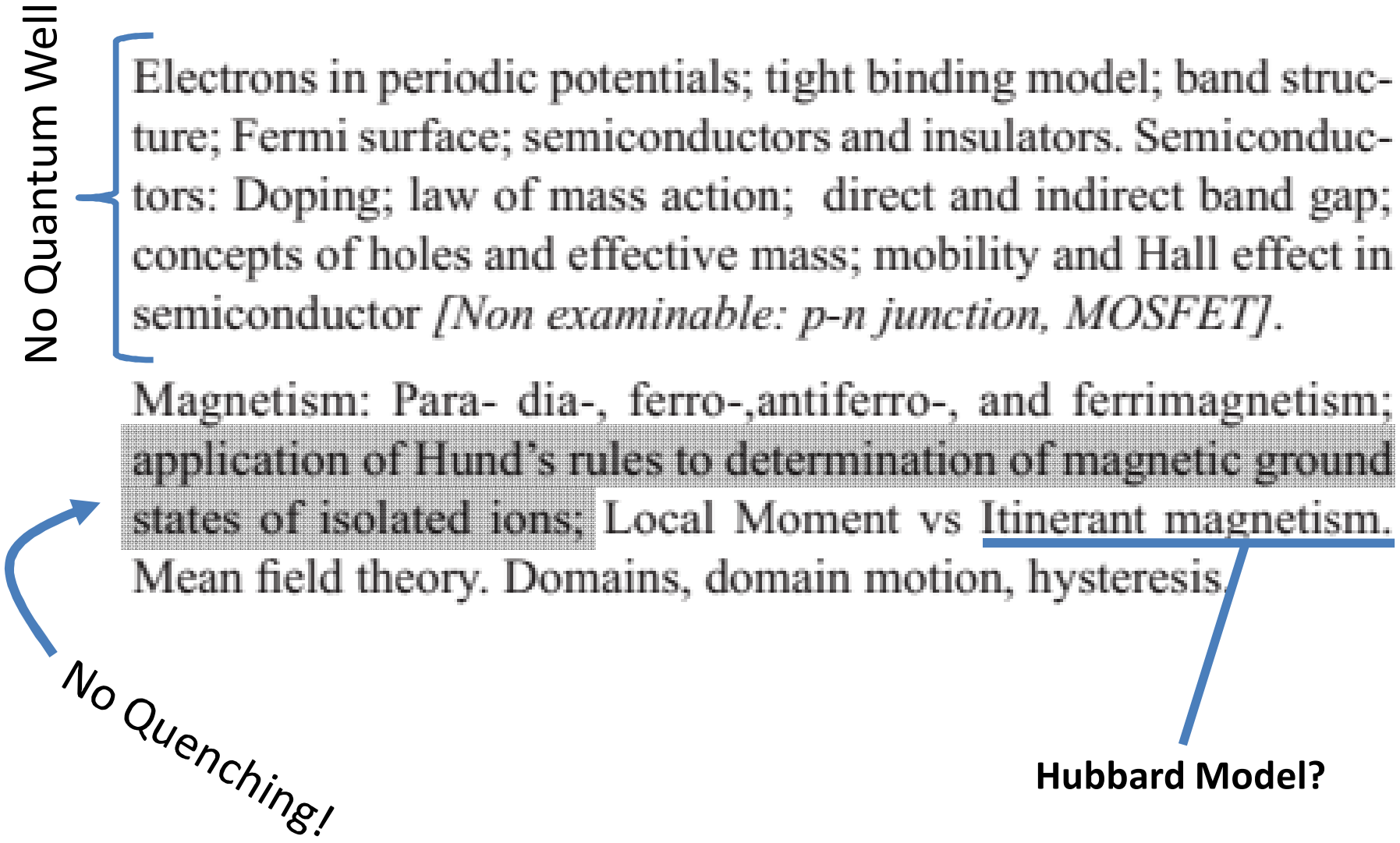
Structure and types of condensed matter. Bonding of atoms: ionic, covalent, van der Waals, metallic *[Non examinable: hydrogen]*. Elasticity and thermal expansion.

Crystals. Bravais lattices, lattice planes, Miller indices and unit cells (conventional and primitive). Reciprocal lattice: Bragg and Laue formulation of diffraction; Brillouin zone; neutron and x-ray scattering.

Electrons in periodic potentials; tight binding model; band structure; Fermi surface; semiconductors and insulators. Semiconductors: Doping; law of mass action; direct and indirect band gap; concepts of holes and effective mass; mobility and Hall effect in semiconductor *[Non examinable: p-n junction, MOSFET]*.

Magnetism: Para- dia-, ferro-,antiferro-, and ferrimagnetism; application of Hund's rules to determination of magnetic ground states of isolated ions; Local Moment vs Itinerant magnetism. Mean field theory. Domains, domain motion, hysteresis.

Syllabus Notes:



General Exam Advice

The Long and Short of it…

Give correct length answers!

Time Limit = 2 hours (0:10+1:50)

Don’t ramble about
something that is not asked!

3. Explain what is meant by the following terms in relation to the electronic bandstructure: *Fermi energy, chemical potential, Fermi surface* and *effective mass.* [4]

Explain how a weak periodic potential in a (one-dimensional) crystal can lead to the formation of a *band gap*. Sketch and describe qualitatively how the band gap and effective masses close to the Brillouin zone boundaries change as the magnitude of the periodic potential is increased. [8]

A fictitious metal crystallises into a simple cubic lattice with lattice constant a and one atom per lattice point. The potential in the crystal is weakly modulated with the periodicity of the lattice. Sketch the first Brillouin zone for this crystal. The metal is monovalent (that is, it has only one valence electron per unit cell). Describe the shape and dimensions of the Fermi surface if the modulation is extremely weak. What if it is somewhat weak (compared to the Fermi energy), but not extremely weak? A second metal has an identical unit cell, but is divalent (two valence electrons per unit cell). Give a qualitative description and sketch of the Fermi surface for this divalent metal. Discuss what happens in both the monovalent and divalent cases as the periodic potential becomes extremely strong. [8]

From Collection (2008 Exam)

From 2001 Exam

5. Explain briefly the origin of the electronic band gap in a typical electrical insulator. [6]

The periodic potential $V(x)$ experienced by an electron in a one-dimensional crystal may be given in the form

$$V(x) = V_0 + V_G\,\mathrm{e}^{-\mathrm{i}Gx} + V_{-G}\,\mathrm{e}^{+\mathrm{i}Gx},$$

where G is the reciprocal lattice vector, and $|V_G| = |V_{-G}|$. Explain why a suitable wavefunction for an electron in such a potential may be written to a first approximation as

$$\psi(x) = A\mathrm{e}^{\mathrm{i}kx} + B\mathrm{e}^{\mathrm{i}(k-G)x}. \qquad [3]$$

By substituting $\psi(x)$ into the Schrödinger equation and comparing coefficients in $\mathrm{e}^{\mathrm{i}kx}$ and $\mathrm{e}^{\mathrm{i}(k-G)x}$, show that the energy of an electron of mass m and wavevector k at the zone boundary is given by

$$E = V_0 + \frac{\hbar^2 k^2}{2m} \pm |V_G|\,.$$

Discuss the significance of each of the three terms on the right-hand side of this equation in terms of band theory. [10]

Using this result explain why diamond is a good electrical insulator, whereas silicon and germanium, which have the same structure type as diamond, are semiconductors. (In the diamond structure there are two tetravalent atoms in the basis.) [6]

From:

Report on B6 (Condensed-Matter Physics) 2014

An additional general comment is that often our students do not know how to answer a physics question: what one generally receives is a stream of formulae with no words, definitions, or linking sentences whatsoever. One has to guess what is in the mind of the undergraduate. I believe it is worth tutors passing on the message to students that such a stream of consciousness does not attract full marks.

Be Clear In What You Are Doing!

(particularly in “Show X” questions)

SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B6. CONDENSED-MATTER PHYSICS

TRINITY TERM 2014

1. State what you understand by the terms *lattice*, *basis*, *structure*, *primitive unit cell* and *conventional unit cell*. [5]

Lattice = an infinite set of points defined by integer sums of a set of linearly independent primitive lattice vectors.

Basis = description of objects in the unit cell with respect to a reference lattice point in the unit cell.

Primitive Unit Cell = A unit cell containing exactly one lattice point

Conventional Unit Cell = A non primitive unit cell which is convenient to work with – usually meaning that it has orthogonal axes.

Structure = ???

= A particular periodic arrangement of atoms

= Lattice + Basis

1. State what you understand by the terms *lattice*, *basis*, *structure*, *primitive unit cell* and *conventional unit cell*. [5]

Write down the coordinates of the lattice points in both the body-centred cubic and face-centred cubic conventional unit cells in terms of their conventional lattice vectors. How many lattice points are there in each of these two conventional cells?

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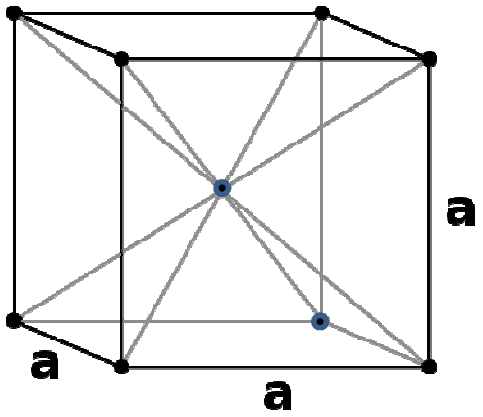
BCC



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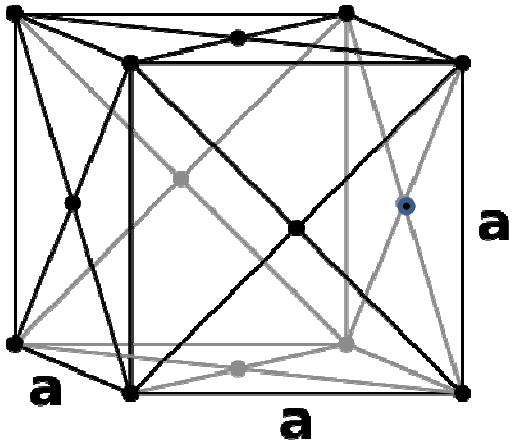
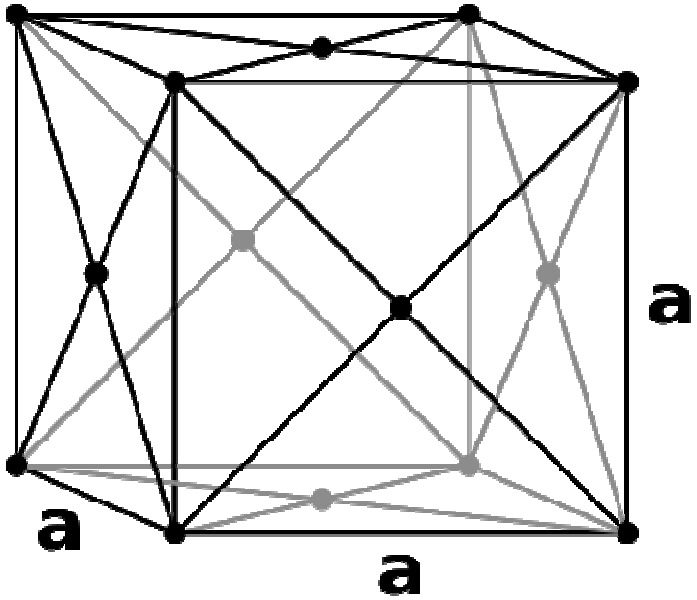


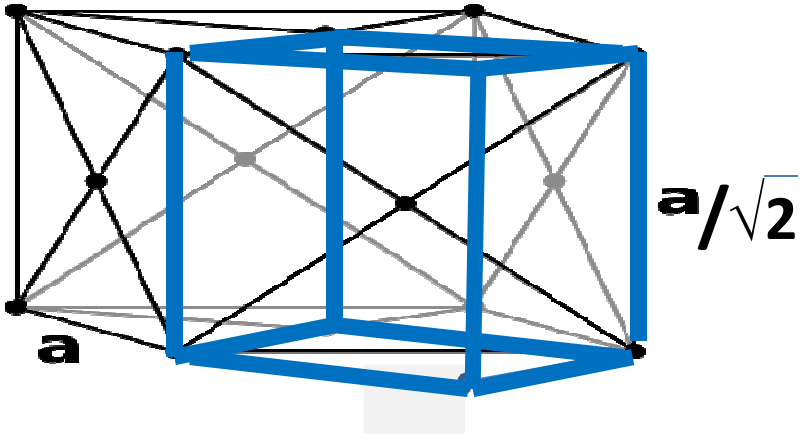
1. State what you understand by the terms *lattice*, *basis*, *structure*, *primitive unit cell* and *conventional unit cell.* [5]

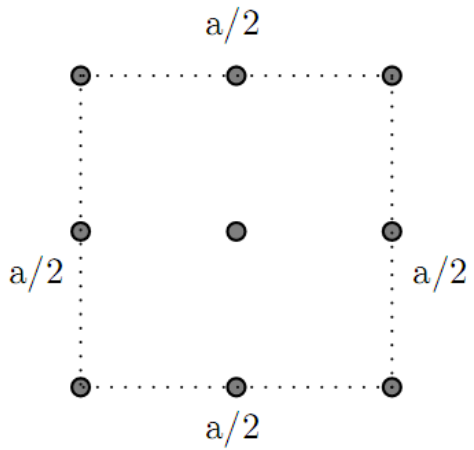
Write down the coordinates of the lattice points in both the body-centred cubic and face-centred cubic conventional unit cells in terms of their conventional lattice vectors. How many lattice points are there in each of these two conventional cells? Without proof, write down an expression for the geometrical structure factor for the X-ray reflections from planes with Miller indices (hkl) of a crystal which contains N atoms in the unit cell. Use your expression to determine the rules governing the reflections that are allowed by the lattice for both face-centred cubic and body-centred cubic lattices. In each case determine what fraction of all possible permutations of Miller indices give rise to allowed reflections [8]

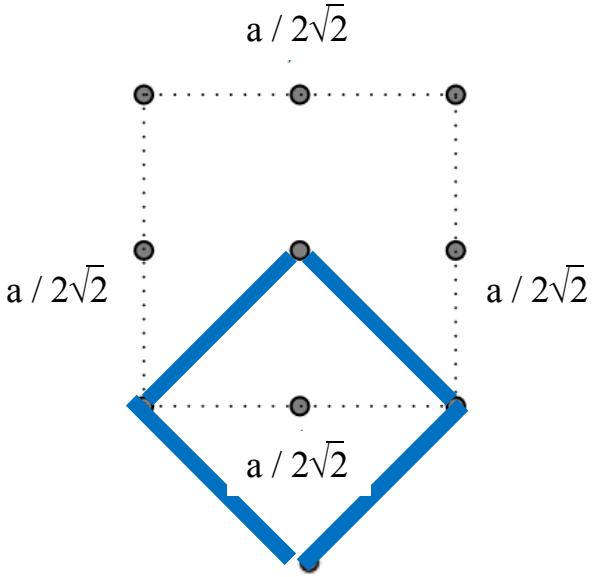
The uniaxial compression of a face-centred cubic crystal, such that the length of the lattice constant along one of its principal axes is reduced, the other two remaining fixed, is known as compression along the Bain path. Show by means of a diagram, or otherwise, that when a face-centred cubic crystal is compressed along the Bain path to the point where its volume is reduced to $1/\sqrt{2}$ of the original volume, it becomes body-centred cubic. [6]

FCC









Unlabeled points at height 0 and a

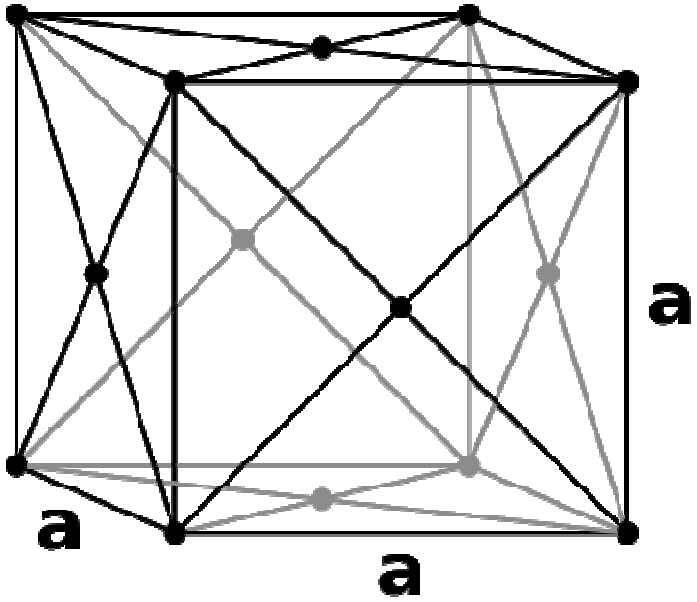
Unlabeled points at height 0 and a/√2

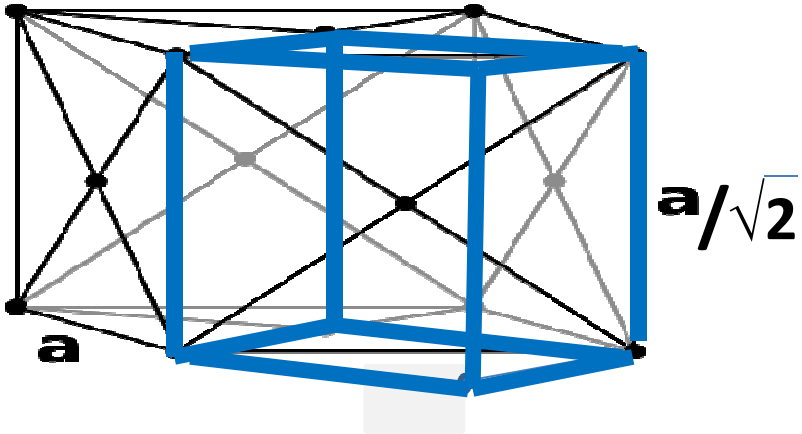
1. State what you understand by the terms *lattice*, *basis*, *structure*, *primitive unit cell* and *conventional unit cell.* [5]

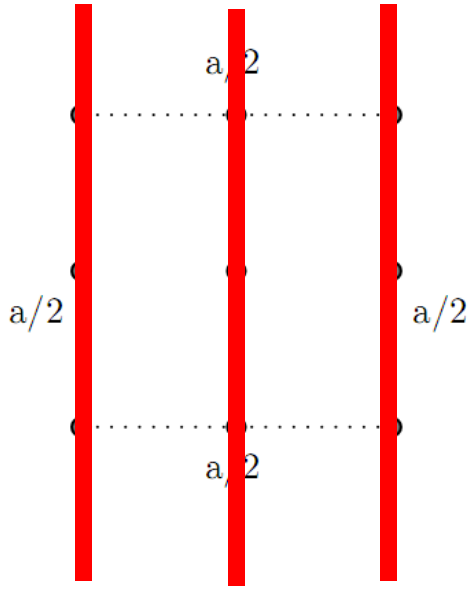
Write down the coordinates of the lattice points in both the body-centred cubic and face-centred cubic conventional unit cells in terms of their conventional lattice vectors. How many lattice points are there in each of these two conventional cells? Without proof, write down an expression for the geometrical structure factor for the X-ray reflections from planes with Miller indices (hkl) of a crystal which contains N atoms in the unit cell. Use your expression to determine the rules governing the reflections that are allowed by the lattice for both face-centred cubic and body-centred cubic lattices. In each case determine what fraction of all possible permutations of Miller indices give rise to allowed reflections, and comment on these fractions. [8]

The uniaxial compression of a face-centred cubic crystal, such that the length of the lattice constant along one of its principal axes is reduced, the other two remaining fixed, is known as compression along the Bain path. Show by means of a diagram, or otherwise, that when a face-centred cubic crystal is compressed along the Bain path to the point where its volume is reduced to $1/\sqrt{2}$ of the original volume, it becomes body-centred cubic. [6]

Using the rules for allowed X-ray reflections, state whether X-ray reflections from the (100), (110), (200), and (111) planes are allowed by the lattice for a face-centred cubic crystal. This crystal is compressed in the z-direction along the Bain path to the point where the lattice becomes body-centred cubic. Consider each of the above four planes to pass through the same set of atoms as the crystal is compressed. What are the Miller indices for each of the planes when written in terms of the lattice co-ordinates referenced to this new body-centred cubic unit cell? Determine which, if any, of the reflections are still allowed, and comment on your results. [6]

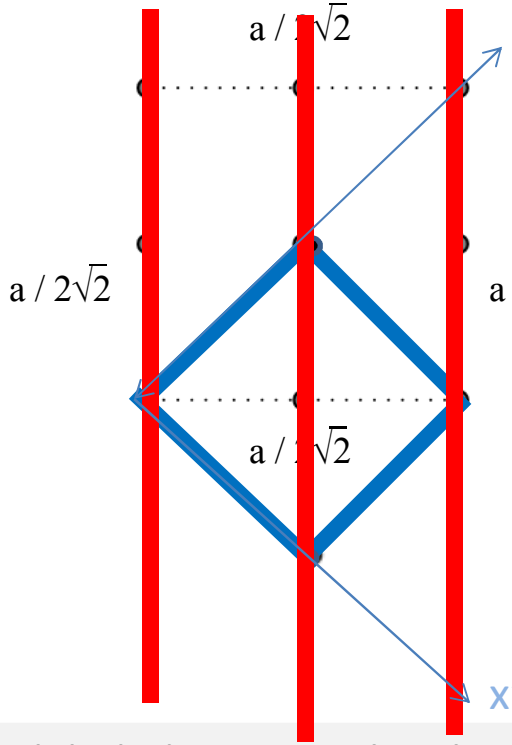






(200)

Allowed reflection for FCC (true recip lat vec)

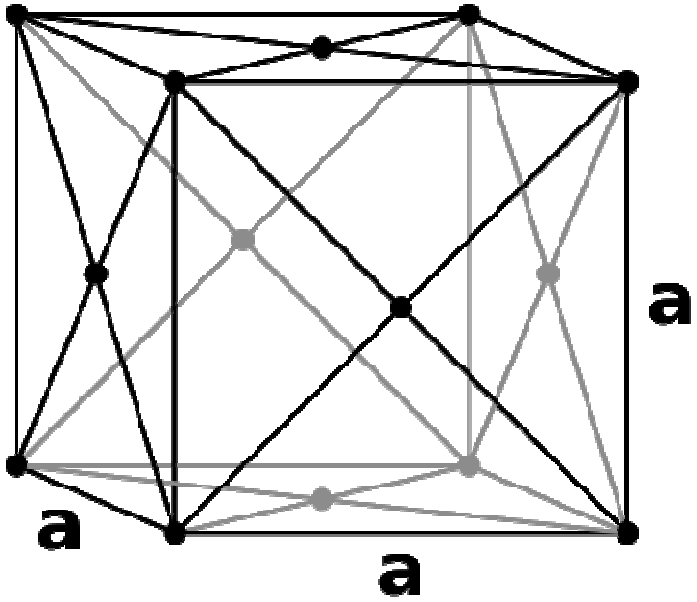


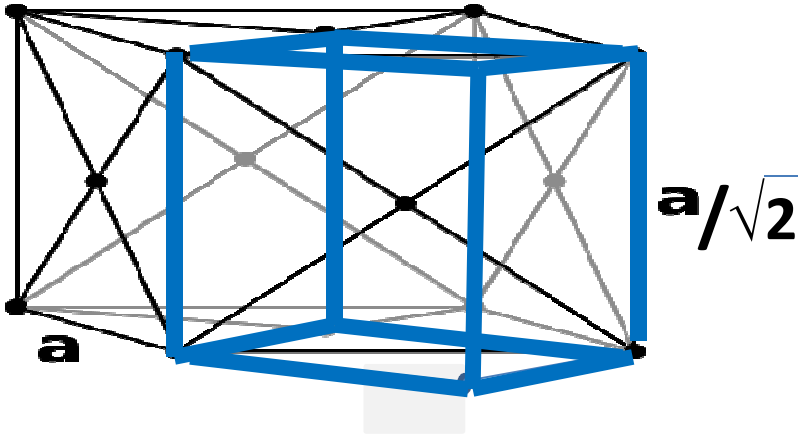
(110)

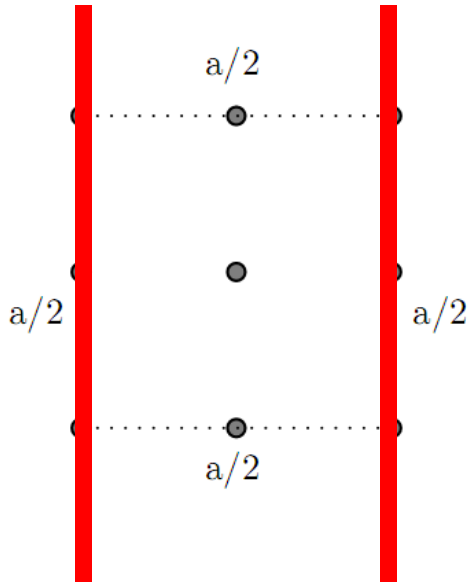
Allowed reflection for BCC

Unlabeled points at height 0 and a

Unlabeled points at height 0 and a/√2

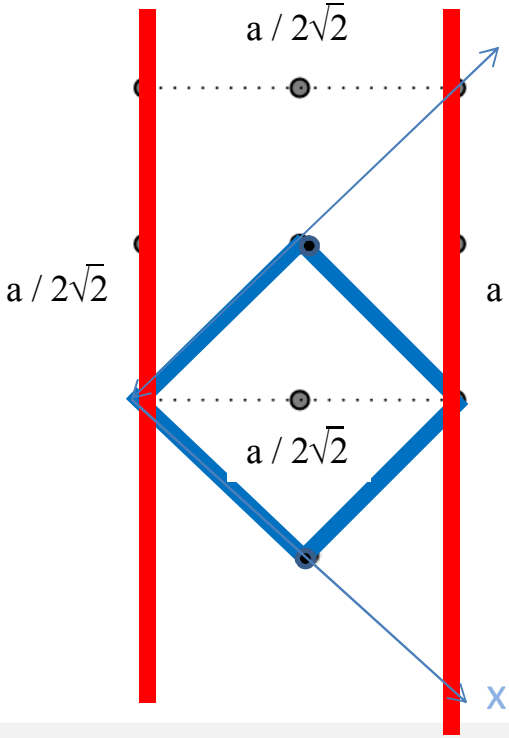






(100)

not allowed
reflection
for FCC

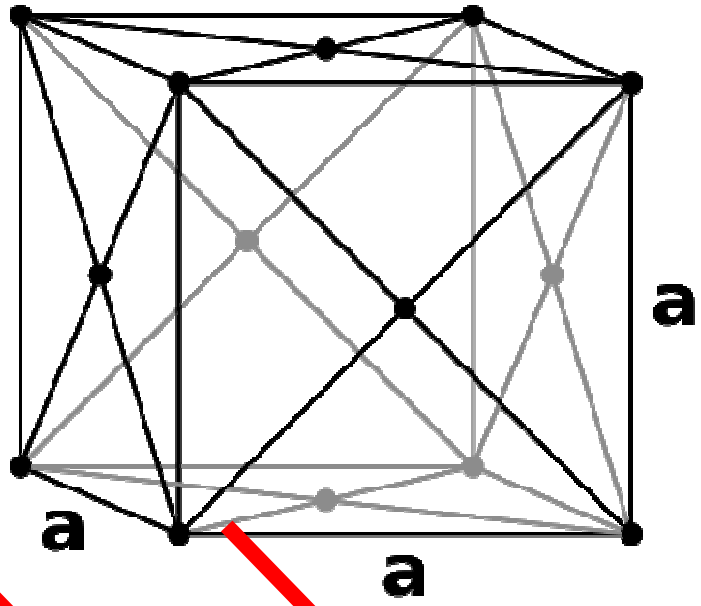


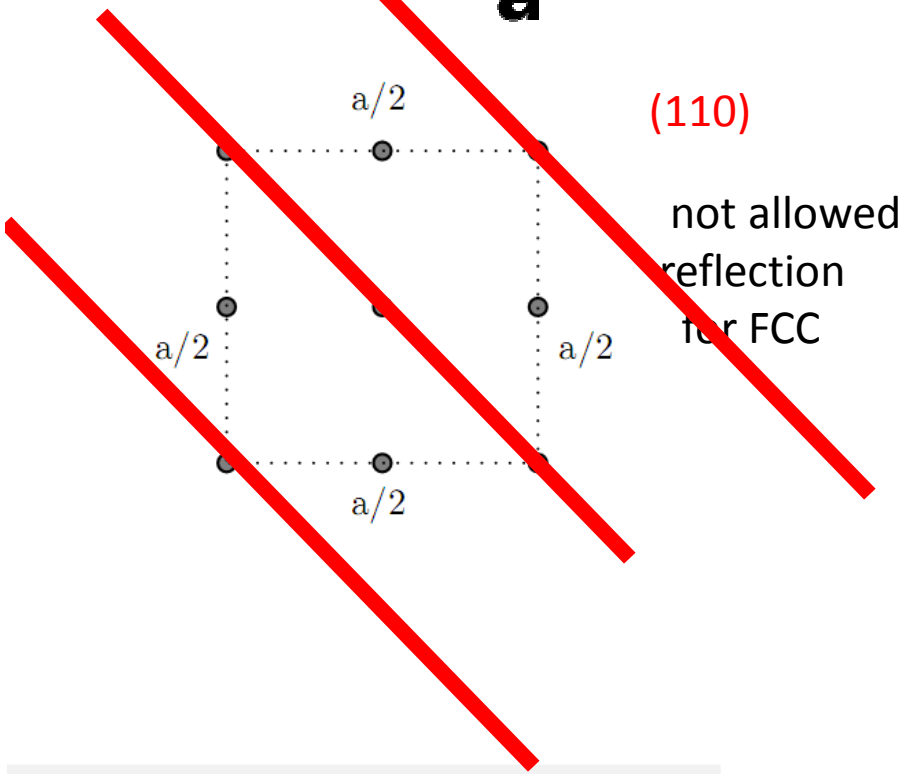
(1/2,1/2,0)

not allowed
reflection
BCC

Unlabeled points at height 0 and a

Unlabeled points at height 0 and a/√2

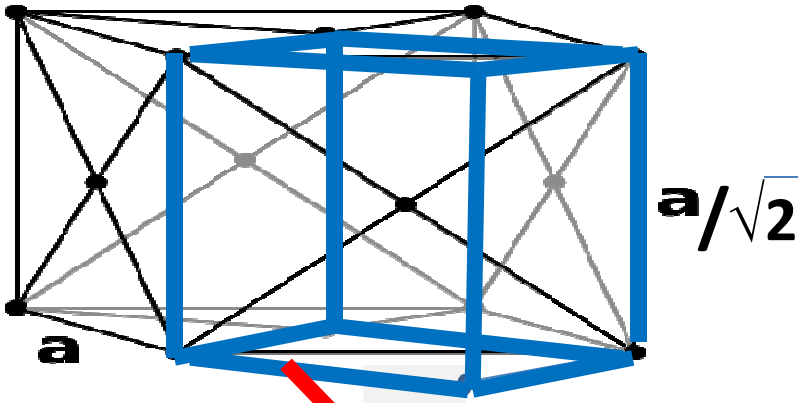


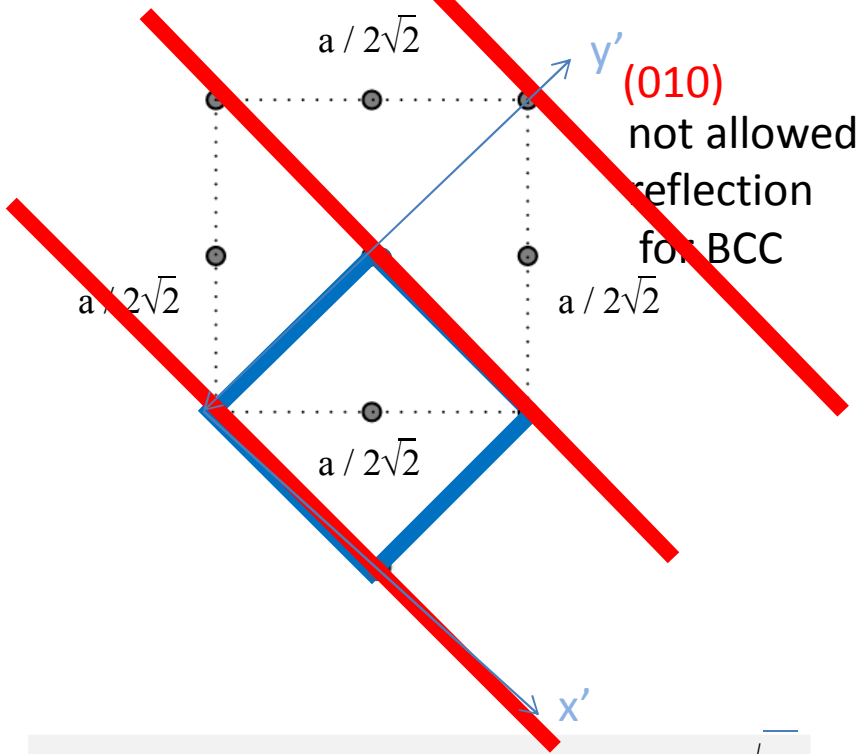


(110)

not allowed
reflection
for FCC

Unlabeled points at height 0 and a

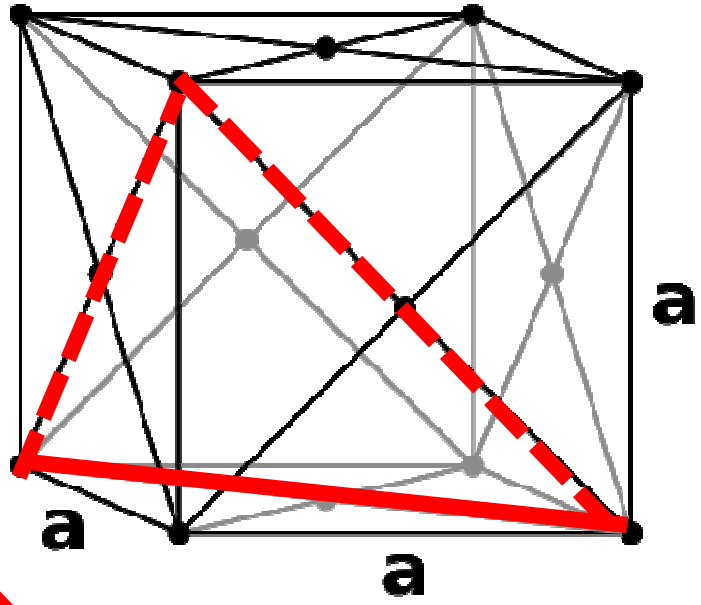


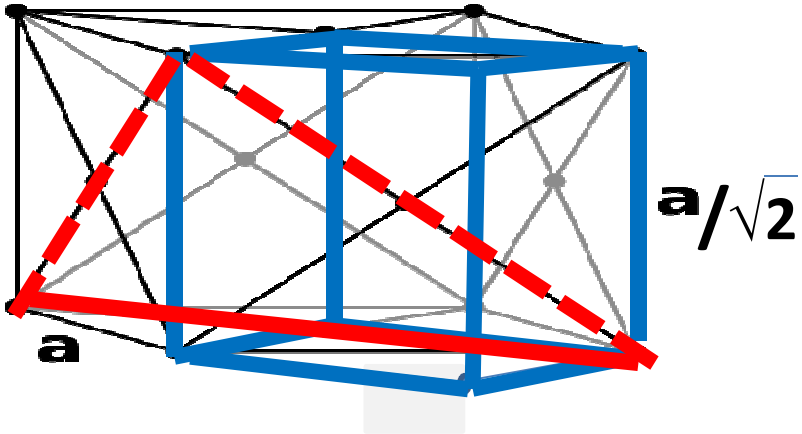


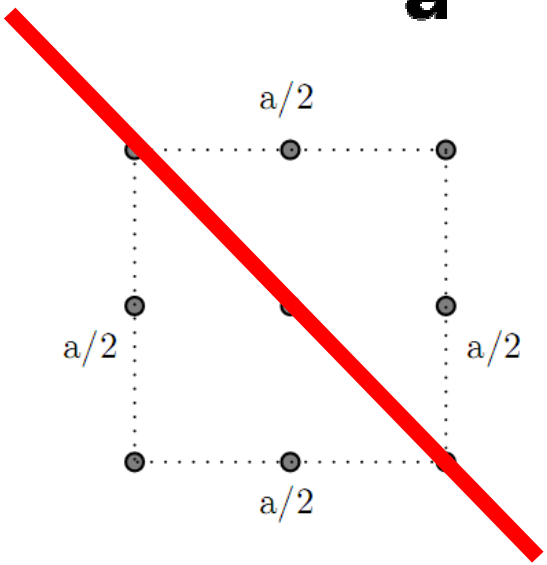
(010)

not allowed
reflection
for BCC

Unlabeled points at height 0 and a/√2

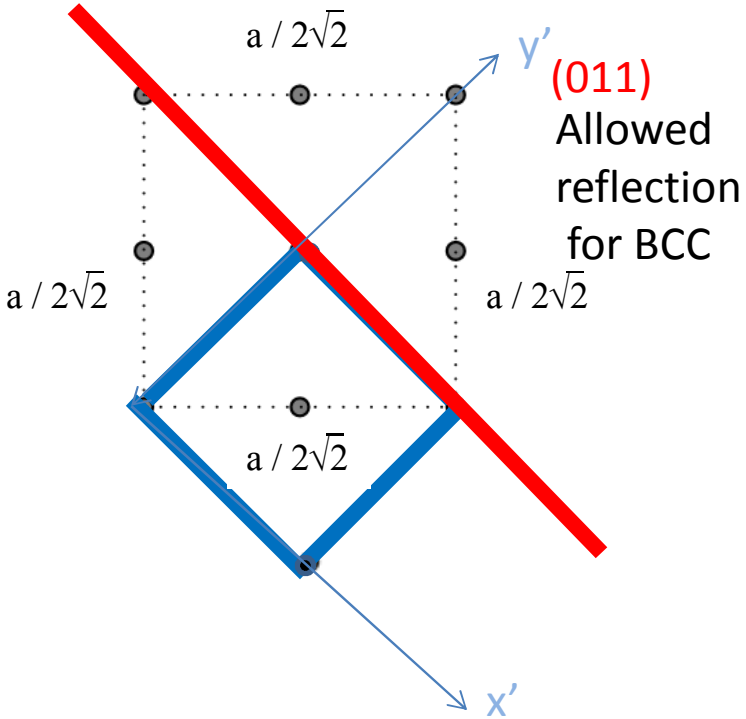






(111)

Allowed reflection for FCC



(011)

Allowed reflection for BCC

Unlabeled points at height 0 and a

Unlabeled points at height 0 and a/√2

1. State what you understand by the terms *lattice*, *basis*, *structure*, *primitive unit cell* and *conventional unit cell.* [5]

Write down the coordinates of the lattice points in both the body-centred cubic and face-centred cubic conventional unit cells in terms of their conventional lattice vectors. How many lattice points are there in each of these two conventional cells? Without proof, write down an expression for the geometrical structure factor for the X-ray reflections from planes with Miller indices (hkl) of a crystal which contains N atoms in the unit cell. Use your expression to determine the rules governing the reflections that are allowed by the lattice for both face-centred cubic and body-centred cubic lattices. In each case determine what fraction of all possible permutations of Miller indices give rise to allowed reflections, and comment on these fractions. [8]

The uniaxial compression of a face-centred cubic crystal, such that the length of the lattice constant along one of its principal axes is reduced, the other two remaining fixed, is known as compression along the Bain path. Show by means of a diagram, or otherwise, that when a face-centred cubic crystal is compressed along the Bain path to the point where its volume is reduced to $1/\sqrt{2}$ of the original volume, it becomes body-centred cubic. [6]

Using the rules for allowed X-ray reflections, state whether X-ray reflections from the (100), (110), (200), and (111) planes are allowed by the lattice for a face-centred cubic crystal. This crystal is compressed in the z-direction along the Bain path to the point where the lattice becomes body-centred cubic. Consider each of the above four planes to pass through the same set of atoms as the crystal is compressed. What are the Miller indices for each of the planes when written in terms of the lattice co-ordinates referenced to this new body-centred cubic unit cell? Determine which, if any, of the reflections are still allowed, and comment on your results. [6]

2. Derive expressions for the Fermi Temperature, T_F, and Debye Temperature, θ_D, of a monovalent metal containing n atoms per unit volume, and within which the speed of sound averaged over polarisations is c. Show that for a face-centred cubic metal with lattice spacing a the ratio of the two temperatures is given by

$$\frac{T_F}{\theta_D} = (6\pi^2)^{1/3}\left(\frac{\lambda}{a}\right) \quad ,$$

where $\lambda = \hbar/(2m_e c)$. [10]

The effective speed of sound in copper (which is a face-centred cubic monovalent metal) is $2700\,\mathrm{m\,s^{-1}}$, and the ratio T_F/θ_D is 240. Calculate T_F, θ_D, and a. [3]

A metal is at a temperature of order θ_D. Within the metal, an electron with the Fermi wave vector, $\mathbf{k_F}$, scatters from a phonon of wave vector $\mathbf{k_{ph}}$ and loses energy. Its new wave vector is $\mathbf{k}'$. Explain why the magnitude of the new wave vector is very close to that of the original wave vector, i.e. $|\mathbf{k}'| = (1-\delta)|\mathbf{k_F}|$, where $\delta \ll 1$. Assuming the phonon obeys the dispersion relation $\omega_{\mathrm{ph}} = ck_{\mathrm{ph}}$ show that

$$\frac{1}{2\delta}\left(\frac{k_{\mathrm{ph}}}{k_{\mathrm{F}}}\right)\frac{1}{k_{\mathrm{F}}} \approx \lambda \quad .$$ [8]

What does the length λ represent? [4]

3. State what you understand by the terms *intrinsic semiconductor*, *extrinsic semiconductor*, *mobility*, and *effective mass*. [4]

Explain what is meant by a *hole* in semiconductor physics, and why it is a useful concept. Give arguments that determine the sign of (i) the effective mass, (ii) the charge associated with the hole. [6]

For the majority of intrinsic semiconductors, the mobility of the electrons is greater than that of the holes. Give a simple argument that explains why this is the case. For pure germanium at room temperature the mobilities of the electrons and holes are 0.36 and $0.18\,\mathrm{m^2\,V^{-1}\,s^{-1}}$ respectively, and the electrical resistivity is $0.5\,\Omega\,\mathrm{m}$. What is the number density of electrons and holes? [6]

A monovalent face-centred-cubic metal with lattice parameter 0.36 nm has a resistivity of $1.7\times10^{-8}\,\Omega\,\mathrm{m}$. Calculate the mobility of the electrons, and comment on the value compared with the mobility of the electrons in germanium. [5]

The two ends of a piece of intrinsic germanium with cross-sectional area $1\,\mathrm{mm^2}$ and length 1 cm are connected to the terminals of a 2 V battery by means of wires made from the above-mentioned metal. The wires have cross-sectional area of $0.5\,\mathrm{mm^2}$. Determine the drift velocity of the carriers in the germanium, and of the carriers in the metal. [4]