2706

SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B6. CONDENSED-MATTER PHYSICS

TRINITY TERM 2014

Monday, 16 June, 2.30 pm – 4.30 pm

10 minutes reading time

Answer **two** *questions.*

Start the answer to each question in a **fresh book.**

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. State what you understand by the terms *lattice*, *basis*, *structure*, *primitive unit cell* and *conventional unit cell.* [5]

Write down the coordinates of the lattice points in both the body-centred cubic and face-centred cubic conventional unit cells in terms of their conventional lattice vectors. How many lattice points are there in each of these two conventional cells? Without proof, write down an expression for the geometrical structure factor for the X-ray reflections from planes with Miller indices (hkl) of a crystal which contains N atoms in the unit cell. Use your expression to determine the rules governing the reflections that are allowed by the lattice for both face-centred cubic and body-centred cubic lattices. In each case determine what fraction of all possible permutations of Miller indices give rise to allowed reflections, and comment on these fractions. [8]

The uniaxial compression of a face-centred cubic crystal, such that the length of the lattice constant along one of its principal axes is reduced, the other two remaining fixed, is known as compression along the Bain path. Show by means of a diagram, or otherwise, that when a face-centred cubic crystal is compressed along the Bain path to the point where its volume is reduced to $1/\sqrt{2}$ of the original volume, it becomes body-centred cubic. [6]

Using the rules for allowed X-ray reflections, state whether X-ray reflections from the (100), (110), (200), and (111) planes are allowed by the lattice for a face-centred cubic crystal. This crystal is compressed in the z-direction along the Bain path to the point where the lattice becomes body-centred cubic. Consider each of the above four planes to pass through the same set of atoms as the crystal is compressed. What are the Miller indices for each of the planes when written in terms of the lattice co-ordinates referenced to this new body-centred cubic unit cell? Determine which, if any, of the reflections are still allowed, and comment on your results. [6]

2. Derive expressions for the Fermi Temperature, T_F, and Debye Temperature, θ_D, of a monovalent metal containing n atoms per unit volume, and within which the speed of sound averaged over polarisations is c. Show that for a face-centred cubic metal with lattice spacing a the ratio of the two temperatures is given by

$$\frac{T_F}{\theta_D} = (6\pi^2)^{1/3}\left(\frac{\lambda}{a}\right) \quad ,$$

where $\lambda = \hbar/(2m_e c)$. [10]

The effective speed of sound in copper (which is a face-centred cubic monovalent metal) is $2700\,\mathrm{m\,s^{-1}}$, and the ratio T_F/θ_D is 240. Calculate T_F, θ_D, and a. [3]

A metal is at a temperature of order θ_D. Within the metal, an electron with the Fermi wave vector, $\mathbf{k_F}$, scatters from a phonon of wave vector $\mathbf{k_{ph}}$ and loses energy. Its new wave vector is $\mathbf{k'}$. Explain why the magnitude of the new wave vector is very close to that of the original wave vector, i.e. $|\mathbf{k'}| = (1-\delta)|\mathbf{k_F}|$, where $\delta \ll 1$. Assuming the phonon obeys the dispersion relation $\omega_{\mathrm{ph}} = ck_{\mathrm{ph}}$ show that

$$\frac{1}{2\delta}\left(\frac{k_{\mathrm{ph}}}{k_{\mathrm{F}}}\right)\frac{1}{k_{\mathrm{F}}} \approx \lambda \quad .$$

[8]

What does the length λ represent? [4]

3. State what you understand by the terms *intrinsic semiconductor*, *extrinsic semiconductor*, *mobility*, and *effective mass*. [4]

Explain what is meant by a *hole* in semiconductor physics, and why it is a useful concept. Give arguments that determine the sign of (i) the effective mass, (ii) the charge associated with the hole. [6]

For the majority of intrinsic semiconductors, the mobility of the electrons is greater than that of the holes. Give a simple argument that explains why this is the case. For pure germanium at room temperature the mobilities of the electrons and holes are 0.36 and $0.18\,\mathrm{m^2\,V^{-1}\,s^{-1}}$ respectively, and the electrical resistivity is $0.5\,\Omega\,\mathrm{m}$. What is the number density of electrons and holes? [6]

A monovalent face-centred-cubic metal with lattice parameter 0.36 nm has a resistivity of $1.7\times10^{-8}\,\Omega\,\mathrm{m}$. Calculate the mobility of the electrons, and comment on the value compared with the mobility of the electrons in germanium. [5]

The two ends of a piece of intrinsic germanium with cross-sectional area $1\,\mathrm{mm}^2$ and length 1 cm are connected to the terminals of a 2 V battery by means of wires made from the above-mentioned metal. The wires have cross-sectional area of $0.5\,\mathrm{mm}^2$. Determine the drift velocity of the carriers in the germanium, and of the carriers in the metal. [4]

4. Define *magnetic susceptibility*, χ_M. Consider a crystal containing N paramagnetic ions per unit volume, each of which has a spin $S = 1/2$. The crystal is placed in a magnetic field of flux density B and is at a temperature T. Show that in the limit of small B, and assuming non-interacting spins,

$$\chi_M = \frac{N\mu_0\mu_{\mathrm{B}}^2}{k_{\mathrm{B}}T} \quad .$$

What is meant by a small magnetic field in this context? [6]

Relaxing the assumption of non-interacting spins, make an estimate of the magnetic field due to one of the ions that is experienced by its neighbour, situated at a distance of order 0.2 nm. Hence estimate the temperature such that the thermal energy of an ion is equivalent to the magnetic interaction energy between neighbouring ions. [5]

The figure shows the inverse magnetic susceptibility as a function of temperature for a different magnetic material that can exhibit a permanent magnetic moment over a certain temperature range. State this temperature range, and outline a simple model that explains the temperature dependence of χ_M for $T \geq T_C$. Given that for certain such materials, $T_C \approx 1000\,\mathrm{K}$, describe a physical mechanism that could account for the magnitude of T_C. [10]

In practice macroscopic samples of the material often display a permanent magnetic moment that is far lower than the theoretical maximum value. Explain why this is the case. [4]