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SECOND PUBLIC EXAMINATION

Honour School of Physics Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B3: VI. CONDENSED-MATTER PHYSICS

TRINITY TERM 2012

Wednesday, 13 June, 2.30 pm – 4.00 pm

Answer **two** *questions.*

Start the answer to each question in a **fresh book.**

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. When studying the scattering of X-rays by atoms, in a simple approximation we assume that the scattered amplitude is proportional to the number of bound electrons in the atom. Why is this a reasonable approximation in most cases? [3]

A collimated beam of X-rays of wavelength 0.154 nm is incident upon a powder sample of NaCl at standard temperature and pressure. The first four diffraction rings correspond to radiation scattered at 27.4°, 31.7°, 45.4°, and 53.8° with respect to the unscattered beam. Determine the lattice type and lattice constant of NaCl under these conditions. [10]

When subjected to a high pressure of 30 GPa the same sample of NaCl undergoes a phase transition (i.e. its crystal structure changes), and it is found to have a simple cubic lattice with a basis of a Na atom at (0,0,0) and a Cl atom at $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$. The smallest angle at which radiation is scattered with respect to the unscattered beam is now 29.7°. Determine the scattering angle of the next diffraction ring, and the number of photons scattered into it relative to the first ring. [8]

What is the volume of the sample at 30 GPa expressed as a fraction of the volume of the sample at atmospheric pressure? [4]

[The atomic number of Na is 11, and of Cl is 17. NaCl is ionically bonded.]

2. Explain the assumptions of the Debye model for the specific heat of a three-dimensional solid. Show that the model predicts a maximum phonon frequency within the solid, ω_D, and obtain an expression for this frequency as a function of the number density of atoms and the speed of sound. [6]

Silver is a face centred cubic crystal with a lattice constant of 0.409 nm, and a Debye temperature of 215 K. Silver can be grown in the form of nanocrystalline cubes where the length of the side of the cube is 180 nm. Calculate the number of phonons present within the nanocrystal at a temperature of 10 K. How might the finite size of the crystal affect the number of phonons present as the temperature is lowered further? [11]

Show that at temperatures large compared with the Debye temperature, the Debye model of the heat capacity predicts that the heat capacity of a solid is $3Nk_{\mathrm{B}}$, where N is the number of atoms. [8]

$$\left[\int_0^\infty \frac{x^2}{e^x - 1}\,\mathrm{d}x \approx 2.404\right]$$

3. Show that in a semiconductor the product of the number density of the electrons, n, and the number density of the holes, p, is given by

$$np = 4\left(\frac{k_{\mathrm{B}}T}{2\pi\hbar^2}\right)^3 (m_{\mathrm{e}}^* m_{\mathrm{h}}^*)^{3/2} \exp(-E_g/k_{\mathrm{B}}T) \; ,$$

where m_{e}^* and m_{h}^* are the effective masses of the electrons and holes respectively, and E_g is the bandgap. [12]

Suggest a suitable dopant to place within Ge to make it into an $n-$type semiconductor, and make an estimate of the energy required for the dopant to be ionized. Compare this energy with the thermal energy of a typical particle obeying classical statistics at room temperature, and comment on your results. [5]

Estimate the minimum concentration of dopant atoms required for Ge to retain extrinsic behaviour up to a temperature of 750 K. At this dopant concentration, what would the hole concentration be at 300 K? [8]

[$\int_0^\infty x^{1/2}e^{-x}\,\mathrm{d}x = \sqrt{\pi}/2$. The (indirect) band gap of Ge is 0.661 eV, $m_{\mathrm{e}}^* = 0.22\,m_{\mathrm{e}}$, $m_{\mathrm{h}}^* = 0.34\,m_{\mathrm{e}}$. The relative permittivity is 15.8.]

4. Explain what is meant by the terms *Wigner-Seitz cell, reciprocal lattice vector,* and *Brillouin Zone.* [3]

Obtain expressions for the Fermi energy and Fermi wavevector of a two-dimensional free electron gas as a function of the number of electrons per unit area. [5]

A two-dimensional crystal has a rectangular lattice with lattice constants a and b ($a > b$). Draw the first two Brillouin zones for the crystal, indicating carefully which parts of reciprocal space are associated with each zone. [6]

Assuming that the crystal is monovalent, and that the potential due to the ions can be neglected, show that the Fermi surface will extend beyond the first Brillouin zone if $a > \pi b/2$. [4]

If the crystal is divalent, and the potential due to the ions is now no longer negligible, explain how the system could be either a metal or an insulator, depending on the strength of the ionic potential. Obtain an expression for the smallest energy gap between the first and second Brillouin zones for the divalent material to be an insulator at very low temperatures. [7]