Some Revision Problems

6.1. **Debye Theory**

Use the Debye approximation to determine the specific heat of a two dimensional solid as a function of temperature. State your assumptions. You will need to leave your answer in terms of an integral that generally one cannot do. At high T, show the specific heat goes to a constant and find that constant. At low T, show that $C_v = KT^n$ Find n. Find K in terms of a definite integral. If you are brave you can try to evaluate the integral, but you will need to leave your result in terms of the Riemann zeta function $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$.

6.2. **Debye Theory II**

Physicists should be good at making educated guesses: Guess the element with the highest Debye temperature. The lowest? You might not guess the ones with the absolutely highest or lowest temperatures, but you should be able to get close.

6.3. **Free Electron Theory**

(a) Explain what is meant by the Fermi energy, Fermi temperature and the Fermi surface of a metal.

(b) Show that the kinetic energy of a free electron gas in 3D is $(3/5)NE_F$ where E_F is the fermi energy.

(c) Consider a two dimensional electron gas. Derive an expression for the density of states.

(d) *Calculate the specific heat at low temperature of this two dimensional electron gas. The following integral may be useful:

$$\int_{-\infty}^{\infty} dx \frac{x^2 e^x}{(e^x+1)^2} = \frac{\pi^2}{3}$$

6.4. **Vibrations I**

(a) Consider a 1 dimensional mass and spring model of a crystal. Write down the dispersion curve $\omega(k)$ for this model (this should be easy by this time). Now write an expression for the specific heat of this 1 dimensional chain. You will inevitably have an integral that you cannot do.

(b)* However, you can expand exponentials for high temperature to obtain a high temperature approximation. It should be obvious that the high temperature limit should give heat capacity $C = k_B$ per atom (the law of Dulong-Petit in one dimension). By expanding to next nontrivial order, show that

$$C/N = k_B(1 - A/T^2 + \ldots)$$

where

$$A = \frac{\hbar^2 k}{6m}$$

where m is the atomic mass and k is the spring constant.

6.5. **Vibrations II** Consider a 1 dimensional spring and mass model of a crystal. Generalize this model to include springs not only between neighbors but also between second nearest neighbors. Let the spring constant between neighbors be called κ_1 and the spring constant between second neighbors be called κ_2. Let the mass of each atom be M.

(a) Calculate the dispersion curve $\omega(k)$ for this model.

(b) Determine the sound wave velocity, Show the group velocity vanishes at the Brillouin zone boundary.

6.6. **Reciprocal Lattice**

Show that the reciprocal lattice of a FCC (face-centered-cubic) lattice is a BCC (body-centered-cubic) lattice. Correspondingly show that the reciprocal lattice of a BCC lattice is an FCC lattice. If an FCC lattice has conventional unit cell with lattice constant a, what is the lattice constant for the conventional unit cell of the reciprocal BCC lattice?

Consider now an orthorhombic face-centered lattice with conventional lattice constants a_1, a_2, a_3. What it the reciprocal lattice now?

6.7. **Scattering**

The Bragg angles of a certain reflection from copper is 47.75° at 20°C but is 46.60° at 1000°C. What is the coefficient of linear expansion of copper? (Note: the Bragg angle θ is half of the measured diffraction (deflection) angle 2θ).

6.8. **More scattering**

KCl and KBr are alkali-halides with the same crystal structure as NaCl: fcc cubic with Na at (0,0,0) and Cl at (1/2,1/2,1/2). KBr shows X-ray diffraction peaks from planes (111) (200) (220) (311) (222) (400)(331)(420), but KCl shows peaks only at (200)(220)(222)(400)(420). Why might this be true?

6.9. **Semiconductors**

Describe experiments to determine the following properties of a semiconductor sample: (i) sign of the majority carrier, (ii) carrier concentration (assume that one carrier type is dominant), (iii) band gap, (iv) effective mass (v) mobility of the majority carrier.

6.10. **More Semiconductors**

Outline the absorption properties of a semiconductor and how these are related to the band gap. Explain the significance of the distinction between a direct and an indirect semiconductor. What region of the optical spectrum would be being studied for a typical semiconducting crystal?

6.11. **Yet More Semiconductors**

Outline a model with which you could estimate the energy of electron states introduced by donor atoms into an n-type semiconductor. Write down an expression for this energy, explaining why the energy levels are very close to the conduction band edge.

6.12. **Magnetism**

Explain briefly the origin of diamagnetism and paramagnetism in atoms.

Consider a crystal of volume V composed of N identical atoms. Each atom has spin 1/2 and $g = 2$. Assume neighboring atoms do not interact, derive an expression for the paramagnetic susceptibility as a function of temperature in the high temperature limit. How would your answer be different if each atom had spin 1?

Explain how this system might be used to make a refrigerator. In reality what limits how well this works?

Discuss what is meant by "quenching" of orbital angular momentum and its consequences for paramagnetism.

6.13. **Mean field theory**

(a) β-Brass is an alloy containing equal numbers of of Cu and Zn atoms. Above a temperature of 730K, the atoms are arranged randomly on a body centered cubic lattice. Below 730K, the lattice becomes simple cubic with Cu atoms largely on the (0,0,0) position and the Zn atoms largely at the (1/2,1/2,1/2) position in the unit cell. The energy of the crystal depends on the occupancy of the sites and is given by

$$E = \frac{1}{2} \sum_{\langle i,j \rangle} J \sigma_i \sigma_j$$

where $\sigma_i = +1$ if the site is occupied by a Cu atom and $\sigma_i = -1$ if the site is occupied by a Zn atom and $J > 0$. Here the sum is restricted to nearest neighbors. Using mean field approximation show that

$$\langle \sigma \rangle = \tanh(\beta z J \langle \sigma \rangle)$$

what is z ? (b) Estimate the magnitude of J (c) Explain, in detail, how this ordering could be observed.