FLUIDS, FLOWS AND COMPLEXITY

SYNOPSIS

Julia Yeomans

A. Navier-Stokes equations

1. Vectors: reminders and identities
2. Continuity equation
3. Material derivative
4. Euler equation
5. Viscosity and the Navier-Stokes equation
6. Comments on the validity of Navier-Stokes
7. Visualising the flow field
8. Solving Navier-Stokes in a simple geometry: channel flow
9. The Reynolds number
(a) non-dimensionalising Navier-Stokes
(b) estimating the Reynolds number
(c) examples of increasing the Reynolds number in different geometries
(d) checking it all works for Poiseuille flow
(e) Dynamical similarity
10. Vorticity
(a) definition and physical interpretation
(b) vortices
(c) the vorticity equation

B. Inviscid flow

1. Kelvin's circulation theorem
2. The dynamics of vortex tubes
(a) time evolution
(b) how are vortices formed?
(c) how are vortices destroyed?
3. Irrotational flow
4. Bernoulli's theorem
5. Lift and drag

C. **Lubrication Approximation**

D. **Waves**

E. **Zero Re hydrodynamics**

F. **Flow instabilities**

G. **Turbulence**

H. **Dynamical Systems**

1. Introduction
2. One dimension
a. Fixed points and linear stability analysis
b. An example - a population model
c. Bifurcations:
(i) saddle node bifurcation
(ii) transcritical bifurcation
(iii) supercritical pitchfork bifurcation + the Rayleigh-Bénard instability
(iv) subcritical pitchfork bifurcation + hysteresis
3. Two dimensions
a. Fixed points and linear stability analysis
b. Classification of fixed points
c. Centres
d. Example 1: the Lotka-Volterra predator-prey model
e. Example 2: the pendulum
f. Limit cycles
g. Hopf bifurcations
h. The Poincaré-Bendixson theorem
4. Higher dimensions and chaos
a. Background
(i) Flows in phase space ...
(ii). ... and the Liouville theorem
(iii) Fractals
b. Lorenz equations
(i) the contraction of volumes in phase space
(ii) fixed points and their stability
(iii) the strange attractor
(v) sensitive dependence on initial conditions
(vi) chaos
c. The logistic map

BOOKS

Elementary Fluid Dynamics D.J. Acheson

Physical Fluid Dynamics D.J. Tritton

Fluid Dynamics for Physicists T.E. Faber

Classics:

Fluid Mechanics L.D. Landau and E.M. Lifshitz

An Introduction to Fluid Dynamics G.K. Batchelor

For dynamical systems:

Nonlinear Dynamics and Chaos S.H. Strogatz