YEAR 2: ELECTRICITY AND MAGNETISM

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Recommended text: David J. Griffiths, Introduction to Electrodynamics, 3rd ed.

REVISION QUESTIONS FROM PROBLEM SET 1: ELECTROSTATICS

1. (a) Calculate the electric field a distance r_0 above the midpoint of a straight line segment of length $2L$ which carries a charge density λ.
(b) Check the limits of your expressions for $r_0 \ll L$ and $r_0 \gg L$ and explain why they are sensible.
(c) For $r_0 \ll L$ rederive the result for the electric field using Gauss' theorem.

2. (a) The electric potential of a spherically symmetric charge distribution $\rho(r)$ is

$$V(r) = A \exp(-\lambda r).$$

Calculate (i) the electric field, (ii) $\rho(r)$.
(b) For a charge distribution $\rho(r) = \epsilon_0 \lambda A \left(\frac{2}{r} - \lambda\right) \exp(-\lambda r)$ calculate (without any reference to part (a)) (i) the electric field, (ii) the electric potential.

3. A metal sphere of radius r_1 is surrounded by a thick concentric metal shell of inner and outer radii r_2 and r_3. The sphere carries charge q and the shell is uncharged.
(a) Find the surface charge density at r_1, r_2 and r_3.
(b) Find the potential at the centre, relative to infinity.
(c) The outer surface is grounded. How do the answers to (a) and (b) change?

REVISION QUESTIONS FROM PROBLEM SET 2: STEADY CURRENTS AND MAGNETISM

1. A particle moves in a region of space where there is a magnetic field B in the x-direction and

an electric field E in the z-direction. At time $t = 0$ it is at the origin with velocity
(a) $\vec{v} = (0, E/B, 0)$,
(b) $\vec{v} = (0, E/2B, 0)$,
(c) $\vec{v} = (0, E/B, E/B)$.
Find, and sketch, its trajectory for each initial condition.

2. A long thin wire carries a current I_1 in the positive z-direction along the axis of a cylindrical co-ordinate system. A thin, rectangular loop of wire lies in a plane containing the axis. The loop contains the region $0 \leq z \leq b$, $R - a \leq r \leq R + a$ and carries a current I_2 which has the direction of I_1 on the side nearer the axis. Find the vector force on each side of the loop which results from the current I_1 and the resultant force on the loop.

3. A loop of wire is formed by two semicircles, $r = a$, $0 < \theta < \pi$ and $r = b$, $0 < \theta < \pi$, joined by radial line segments at $\theta = 0$ and $\theta = \pi$. Find the magnetic field at the origin when the wire carries a current I anticlockwise.

4. (a) Show that the magnitude of the magnetic field on the axis of a solenoid closely wound with n turns per unit length and carrying a current I is

$$B = \frac{1}{2}\mu_0 n I(\cos\phi_1 - \cos\phi_2).$$

Use a diagram to define ϕ_1 and ϕ_2 and to show the direction of the field.
(b) Hence show that the magnetic field on the axis of a long solenoid at the ends is half the value at any point within an infinite solenoid with the same n and I.

5. Consider a round straight wire carrying a current density J throughout, except for a round cylindrical hole parallel to the wire axis of constant cross section. Call the radius of the wire R, the radius of the hole R_H and the distance of the centre of the hole from the centre of the wire a. Take $R_H < a < R$ and $R_H < R - a$. Calculate and sketch the magnetic field $\vec{B}$ as a function of position along a radial line through the centre of the hole.