Renormalisation Group & Critical Phenomena   Hilary 2005   Prof. J. Cardy

Problem Set 2

The solutions to these problems should be handed in by **Monday, April 11** for marking. They should be sent to Dr. P. Calabrese in Theoretical Physics. They will be reviewed at a problems class at a time TBA, which will be conducted by Dr. Calabrese. **Please put your e-mail address on the homework so we can notify you of the time and place of the problem class.**

These problems are all from the book and are numbered accordingly. Hints for doing them can be found on my web page, but try them without first. Any other questions before the due date may be addressed to me at j.cardy1@physics.ox.ac.uk.

4.4 Consider a three-dimensional Ising model in a slab geometry, with finite thickness L, but effectively infinite in the other two directions. The ultimate critical behaviour of this system should be two-dimensional, with a specific heat singularity of the form $A(L)\ln|T - T_c(L)|$. How should $T_c(L)$ and $A(L)$ depend on L for large L?

5.1 Consider the continuous spin tricritical Ising model, in which the dominant interaction is proportional to S^6. Show that the upper critical dimension is $d_c = 3$, and calculate the RG eigenvalues to first order in $\epsilon = 3 - d$, using the same method as discussed in the lectures. [Note that there are two thermal eigenvalues to be calculated.] How does the specific heat behave in three dimensions?

5.4 What are the logarithmic corrections to the dependence of the correlation length on the reduced temperature t for the $O(n)$ model in $d = 4$? Check this by comparison with an explicit calculation in the limit $n \to \infty$.

5.5 The operator which breaks the $O(3)$ symmetry of the Heisenberg model down to uniaxial symmetry is proportional to $S_z^2 - \frac{1}{2}(S_x^2 + S_y^2)$. Calculate its RG eigenvalue, and hence the corresponding cross-over exponent, to first order in ϵ.

6.1 Calculate the two-point function, and hence the scaling dimension and the renormalization group eigenvalue of the operator $\cos p\theta$ in the Gaussian model in two dimensions. Show that it is irrelevant above some critical temperature T_p. What is the nature of the low temperature phase $T < T_p$? Now consider such a perturbation on the full XY model, with vortices included. Show that for sufficiently large p, the RG implies that this model should have two phase transitions as the temperature is varied.