Introduction to Conformal Field Theory Trinity Term 2014 Prof. J. Cardy

Exercises 1

These problems will not be marked. Solutions will be posted on my web page in due course. Any questions or clarifications before then can be addressed to me at `j.cardy1@physics.ox.ac.uk`

1. Using conformal invariance, show that the 2-point function $\langle\phi_1(r_1)\phi_2(r_2)\rangle$ of two different primary fields in R^d vanishes unless their scaling dimensions are equal.

2. Using a (conformal) stereographic projection, compute the 2-point function of a CFT on the sphere S^2 with the standard metric in spherical coordinates.

3. In 2d, use the conformal mapping $z \to w = (\beta/2\pi)\log z$ to compute the 2-point function on a cylinder of circumfererence β. By thinking of the coordinates along and around the cylinder as space r and imaginary time τ, this corresponds to the 2-point function of a 1+1-dimensional CFT at finite temperature β^{-1}. By analytically continuing $\tau \to it$, compute the form of the time-dependent 2-point function at finite temperature.

4. In the upper half z-plane, with a suitable boundary condition on the real axis, scale covariance implies that a 1-point function in a CFT behaves like $(\mathrm{Im}\, z)^{-x}$ where x is the scaling dimension. What is the form of the 1-point function in the interior of (a) the disc $|w| < R$; (b) an infinitely long strip of width L?

5. The action for a free Majorana fermion is

$$S = \int (\psi\partial_{\bar z}\psi + \bar\psi\partial_z\bar\psi)d^2z$$

where $(\psi, \bar\psi)$ are fields satisfying the usual rules of Grassmann integration. Compute the stress tensor T, show that its OPE with ψ has the expected form, and evaluate the value of the central charge c from the 2-point function $\langle TT\rangle$.