Quantum Matter 2, Problem Sheet 2

1 Entanglement growth after a quantum quench

Consider a tight-binding model of free, spinless fermions with Hamiltonian

$$H = -J\sum_{j=1}^{L} c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1} \ , \qquad J > 0. \tag{1}$$

The system is initialized in the product state

$$|\Psi(0)\rangle = \prod_{j=1}^{L/2} c_{2j}^\dagger |0\rangle. \tag{2}$$

Determine the correlation matrix of the system in the thermodynamic limit as a function of time after the quench. Use this result to numerically compute the bi-partite entanglement entropy of a subsystem consisting of ℓ neighbouring sites $S_\ell(t)$. What is the asymptotic value reached at late times for large values of ℓ? Explain.

2 Interaction effects after quantum quenches

Consider a model of interacting, spinless fermions with Hamiltonian

$$H = -J\sum_{j=1}^{L} c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1} + \lambda \sum_j n_j n_{j+1} \ . \tag{3}$$

(a) Using symmetry arguments show that the momentum space Green's function can be written in the form

$$G(p,q,t) = \langle \Psi(t)|c^\dagger(p)c(q)|\Psi(t)\rangle = g_+(p,t)\delta_{p,q} + g_-(p,t)\delta_{q,p+\pi}. \tag{4}$$

(b) Determine the equations of motion for the momentum space single-particle Green's function after a quantum quench from the state (2) using time-dependent, self-consistent mean-field theory. Describe in detail how you would solve these equations numerically.

(c) Show that the total energy is conserved in time-dependent, self-consistent mean-field theory.

Hints: Decouple the interaction term in *position space* introducing an appropriate number of time-dependent mean fields. This results in a quadratic Hamiltonian, from which you can obtain the equations of motion for $c(p,t)$ and hence for $G(p,q,t)$.

3 Lindblad equation for spinless fermions with particle loss

Consider a Lindblad equation with Hamiltonian

$$H = -J\sum_{j=1}^{L} c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \ , \quad \{c_j, c_k^\dagger\} = \delta_{j,k}, \tag{5}$$

and jump operators

$$L_j = \sqrt{\gamma} c_j \ . \tag{6}$$

(a) Derive the equations of motion for the Green's functions

$$g(j, k, t) = \mathrm{Tr}\left[\rho(t) c_j^\dagger c_k\right] \ , \qquad f(j, k, t) = \mathrm{Tr}\left[\rho(t) c_j c_k\right] . \tag{7}$$

(b) Solve them (analytically or numerically) for an initial density matrix

$$\rho(0) = |\Psi(0)\rangle\langle\Psi(0)| \ , \qquad |\Psi(0)\rangle = \prod_{j=1}^{L/2} c_{2j}^\dagger |0\rangle. \tag{8}$$

and plot the time evolution for $g(2j, 2j, t)$, $g(2j+1, 2j+1, t)$ (why these?) and $f(j, \ell, t)$. Comment on your results.

(c) What is the steady state density matrix? Prove it (under the assumption that there is a unique steady state).