Strategy for solving Statistical Mechanics Problems

- Canonical distribution: fixed number of particles in equilibrium at temperature T

  Probability $p_j \propto e^{-\beta E_j}$, $\beta = 1/k_B T$

- Partition function: $Z = \sum_j e^{-\beta E_j}$,

- Thermodynamic functions:

$$U = -\frac{\partial \ln Z}{\partial \beta}, \quad S = -k_B \sum_j p_j \ln p_j, \quad F = -k_B T \ln Z, \quad \text{etc.}$$

- Equipartition theorem: In a classical system, the mean energy per quadratic degree of freedom is $\frac{1}{2}k_B T$

- $Z_N = (Z_1)^N$ — distinguishable particles

  $= (Z_1)^N/N!$ — indistinguishable particles

- Grand canonical distribution: variable number of particles in equilibrium at temperature T

- Grand partition function $\mathcal{Z} = \sum_j e^{-N_j \beta (E_j - \mu)}$,

$$f(E) = \frac{1}{e^{\beta(E-\mu)}+1} \quad \text{fermions}$$

$$f(E) = \frac{1}{e^{\beta(E-\mu)}-1} \quad \text{bosons}$$